Writing to Learn Mathematics

Bernadette Russek

Contrary to popular belief, mathematicians must write and must write well. Statisticians write coherent reports; math educators express themselves in the discipline; and pure mathematicians communicate complex, yet precise, ideas. These forms of writing are addressed in the courses offered in the Math Department. However, there are also other motivations for using writing in math classes. Writing is a valuable assessment tool. It is used to assess attitudes and beliefs, mathematics ability, and ability to express ideas clearly. It can be used for student reflections on their own work, such as in the creation of a portfolio. It is used as a tool for students to investigate topics in The History of Mathematics, such as Women in Mathematics, or the Chronology of Pi. It is used to open doors of communication with students who may have math anxiety or who have "I hate math!" feelings, students who may have never really 'spoken' to their mathematics professor before. There is a wide variety of use of writing in the Mathematics Department. Few teach writing per se, most use it in context of mathematics learning. However, most faculty do expect well-organized thoughts, good grammar, and clear communication.

Some of the tasks fall into the category of writing about mathematics learning. For example, in Theory & Methods of Mathematics Learning, on the very first day students arc asked to write a paragraph on *What is Mathematics?* (see appendix for a

sample response) or *What Makes an Effective Math Teacher?* Not only do students use writing throughout the course to learn about mathematics education, but they also learn about the role of writing in mathematics learning. In this course they scrutinize the English language and examine how it communicates mathematical ideas.

In Introduction to Algebra, a number of writing tasks are assigned. Donna Kelly gives a written assignment every week. She presents a math 'trick' on Monday and the students have to write why it works in a couple of paragraphs that are due on Friday. She requires complete, coherent sentences, and stresses that students check spelling and grammar. The assignment is graded mainly on content; however, she does take off points for incoherent, incomplete sentences and excessive spelling errors.

In Introduction to Algebra, Part I, Bernadette Russek provides a set of writing prompts (see appendix) to students at the beginning of the semester, and every so often asks for one of these to be passed in. The first writing assignment is an effort to get to know the students and establish communication links. The prompt given states: "Write a 'mathography' in which you describe your feelings about and experiences in mathematics, both in and out of school. Include the completion of the statement: *What I like most (or least) about math is . . .*" Another prompt is, "Write a letter to a classmate who could not attend class today so that she/he will understand what we did and learn as much as you did. Be as complete as possible." The one she likes best is, "Write a letter of advice to a student who is going to take this class next year." The students like to do these assignments; they like the dialogue and the opportunity to express their thoughts about an oftentimes mystical and threatening world of math.

In Problem Solving, Donna Kelly assigns problems to groups of students. These problems are mini-research problems that the group must present to the class. In addition to their presentation, the group must submit a written report including the original data, table of values, graphs, interpretation of data, notice patterns, and

observations. The report must be in complete, coherent sentences, have correct spelling and grammar. Again, this assignment is graded mainly on content, but points are assigned for the writing requirements.

In Number Systems, a course designed for Childhood Studies majors, Marilyn Wixson requires a Text Review and an Essay. The essay asks the students to reflect upon one or two of the lab activities. It is to be two or three type-written pages long, and must include at least two outside references with appropriate footnotes. The essay should address the following:

1. Identification and description of at least two mathematical concepts illustrated by each activity.
2. An explanation of what you would do to the activity so you could use it with students at two or three levels- elementary, middle or secondary.
3. A detailed description of some other activities you could augment or use to replace the given activity in the teaching of the concepts identified.

In Problem Solving, Bernadette Russek requires a portfolio, a long-term project of student-selected work. Portfolios are rich in student reflection statements (see appendix). Students must justify the selection of each piece. They must also include a letter to the reader establishing the student's math background, a "banner statement," and organize the selections into a meaningful whole. There is no attempt to teach writing in this assignment; writing is used as an assessment tool.

Writing is an important part of The History of Mathematics course. Discussion questions are on the exams. For example, "Describe the accomplishments of Fibonacci. Write in complete sentences with full explanations." Students in this course must also write a term paper on some mathematician or mathematical discovery or phenomenon, such as Cryptography or "Perspective drawing as developed by the Renaissance artists." Paul Estes' instructions for this assignment include concern for correct writing:

This final product should be the culmination of

searching for information, reading, organizing your thoughts, writing a first draft, and revising into finished form with careful attention to writing basics (correct grammar, sentence structure, spelling, etc.) and thorough proofreading. Credit for your sources should be given in one of the standard formats as prescribed in Diana Hacker's *Writer's Reference* or some other writer's handbook.

As we move into more mathematical courses, writing continues to be of importance. Both statistics courses expect a level of verbal analysis and written communication of findings. Questions sound like, "Describe the overall pattern of...;" "Explain why...;" "Justify your statements." Jon Maatta assigns case studies with a lot of reading and a lot of writing {as well as a lot of statistical analysis). He asks questions such as, "Investigate each of the variables in this problem and comment on anything that is unusual" ; "... interpret the resulting intervals. You should include an interpretation for both the prediction and confidence intervals for at least one of the months for a particular degree day;" "What can you do to improve this model within the limitations that Harold faced? Be explicit by designing possible models that might improve our ability to predict gas consumption." These kinds of questions demand that the student express the situation clearly via the English language.

The course Geometries provides a number of opportunities to write. Of course, there are a number of proofs that must be written out clearly and logically. Furthermore, it is a course with a number of projects, which demand writing. For computer projects, students are asked to keep a log of their progress. The Computer Log asks that they describe:

- New skills necessary to do this project
- Sticking points (problems with the software, or understanding the assignment, or printing, or whatever)
- What I learned.

Students are asked to think about questions to help organize

their thinking as they go through the problems. These questions are:

- Do I understand what is being asked? Do I understand all the terms?
- Have I answered each part in the way the question asks?
- Are all my conjectures supported by some stated evidence?
- Is my work organized and presented in a clear and readable manner?
- Are my answers clear to an outside reader?
- Could I verbally summarize the general conclusions?

Throughout the Geometries course students are asked to "describe," "compare," "investigate," "explain." This kind of question requires an answer in written form. It encourages students to think about their thinking and to better understand that mathematics is more than a lot of short symbolic answers.

In the Elementary Functions course, which is pre-calculus mathematics, Enid Burrows also expects various forms of writing to learn mathematics. Students are expected to submit an email synopsis of the reading material on a regular basis. In a paragraph submitted on email, they are to describe their understanding of the text section to be discussed in that day's lesson. In this course, the text also encourages student writing in *Writing to Learn* sections of the homework sets. For example, "Solve the equation three times—once algebraically, once graphically, and once numerically. Describe the advantage and disadvantage of each method." This kind of exercise is a substantial break from the traditional method of teaching mathematics.

Finally, in the more esoteric courses, such as Algebraic Structures, Norm Cote contends that there is a need to read and write precisely mathematical text, such as proofs. Critical analysis of proofs provides a model for students to then write their own mathematical proofs clearly and logically. In this course, mathematics is a language and is used to express ideas, but we use the English language as a basis for this communication. Students are

taught to use both languages to communicate their ideas.

In conclusion, all mathematics courses expect a degree of writing. It is routine to observe direct use of writing in note taking, making summaries, and descriptions. It is routine to observe linguistic translation, the translation from mathematical symbols to words, and the translation from the English representation into mathematical symbols and equations. It is also standard practice to write persuasive proofs. What is somewhat different today is the use of writing in the form of journals, paragraph descriptions, and explanations to enhance learning and as part of the assessment process. Instructors ask students how they feel about the mathematics that they are learning, to enter into a dialogue with the teacher, and to reflect on their work and their readings. This is a new direction in writing to learn mathematics.

Appendix

Example of an MA107 Assignment Sheet for Journal Writing

Date:

a. Write a letter to a classmate who could not attend class today so that she/he will understand what we did and learn as much as you did. Be as complete as possible.

b. Reflect on your participation in class today and then complete the following statement. Select one of your choice.

I learned that I ...
I was surprised that I ...
I discovered that I ...

I was pleased that I ...

c. Reflect on where you are in the course and complete the following statements. Select two.

Now I understand _____

I still do not understand _____

I can help myself by doing _____

You can help me by _____

d. Write a "mathography" in which you describe your feelings about and experiences in mathematics, both in and out of school. Include the completion of the statement: What I like most (or least) about math is ...

e. Write a letter of advice to a student who is going to take this class next year.

f. Explain to a high school senior why it is important or not important to do mathematics.

g. Design two mathematical bumper stickers, one funny and one serious.

Examples of Student Writing in Mathematics Learning
Responses to the MA107 Assignment Sheet for Journal Writing

part a:
Dear Classmate,

Today was not a good day to miss because we went over Scientific Notation. Scientific notation is a system used that makes very big #'s and very small #'s easyer (sic) to see and write. For example, 72,000,000 = 7.2 x 107, because if you did (this) out you would get 72,000,000. It's just nicer. Make sure you get to class next time.

part ***c:***
Now I understand the problems that involve charts. At first I had trouble with the coin, stamp, and Integer problems. After reading the corresponding text, which I read slowly and thoroughly to make sure I absorbed every bit of info., I began the homework. I breezed right through it. I find it much easier to do all the reading before I start the work.

part **c:**

When I do these problems containing fractions, I still don't understand how to make them whole numbers.

You can help me by doing more math problems w/ fractions.

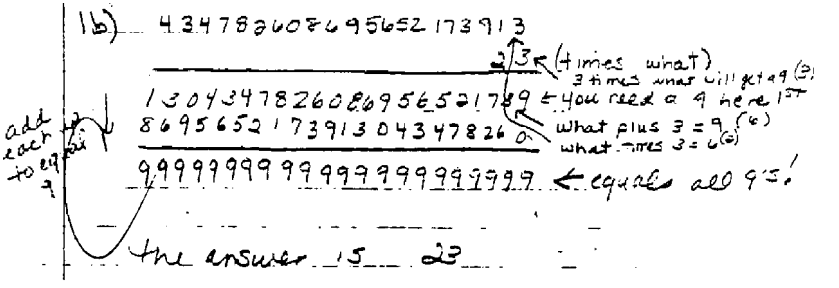
* * *

Excerpts from a Portfolio:

An example of an "affective" endeavor, such as risk taking perseverance, willingness to make various attempts, positive attitudes, change in attitudes, etc.

What number times 4347826086956521739 13 gives all 9's for

an answer? This problem was one of our tests, and it was the hardest problem for me to figure out because every number had to be kept in a straight line and very organized. One mistake meant the whole problem was wrong. As I repeatedly tried to do this problem, I had to keep copying it over to start over. When I copied over the number at one point, I made a mistake and forgot a number. So, I had an 18 digit number for the answer, which didn't look right to me, and I later figured out that the solution was only two digits long. Consequently, I had made the problem harder than it was, and it made me redo the problem for about six hours all together, but I had to take risks and I had to persevere to get the right answer. My attitude towards the whole problem had to be adjusted by looking at the problem differently and trying to figure out where I went wrong.



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Before and After Taking MA401-Theory and Methods of Math Learning

"What is Mathematics?" by Melissa
(before taking course) 1/28/98

Mathematics to me is scary. I am not strong in math at all, so I have negative feelings towards it. Part of my problem is that I

refer to math as a science and I also dislike science. Although, I'd have to admit that my opinion is gradually changing through the years. It seems to make more sense as you play with it more and have more patience. Math, to me, is a bunch of numbers and symbols used to represent different relationships and measurements. I immediately think of problem solving and reasoning. Math is used to solve all sorts of problems and to think out solutions to computations. Math is a way to communicate with numbers and symbols. So it is just like the English language, but on a different level. There are many different kinds of math: algebra, geometry, finite, trigonometry, statistics, calculus, and so on. They each focus on a different topic in math. I prefer the easier stuff like algebra and geometry. I can eventually make sense and figure it out, unlike trig and stats. I like to have a lot of repetition and many different examples to look at. When it comes to math I am a slow learner, as well as a visual learner. I need to see all the steps written out. The math that is taught in the primary grades is easy to understand. I do not think I will have a problem teaching it because I know what children may have problems with because I have experienced some of the same problems myself.

(after taking course) 4/30/98

My opinion has definitely changed since the beginning of this course. I learned not to be afraid of math because there is nothing to be afraid of. I learned how children think so now I know how to teach math. I also have a better idea of what I need to teach. I have grown to appreciate math more now because I learned how easy it is to integrate it with language arts, science, and much more. Integrating it makes it more fun to work with.